

PHENOMENOLOGICAL THEORY OF FRICTION IN THE QUASISTATIC LIMIT : COLLECTIVE PINNING AND MEMORY EFFECTS

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Abstract

When a elastic body is moved quasistatically back and forth over a surface, the friction of the interface is experimentally observed to circulate through a hysteretic loop. The asymptotic behaviour of the hysteresis loop is approached exponentially. We describe how this behaviour is connected to the collective properties of the elastic instabilities suffered by the elastic body as it is displaced quasistatically. We express the length scale of the exponential in terms of the elasticity of the surface and the properties of the rough substrate. The predicted scaling are confirmed numerically.

Résumé

Lorsqu'un mouvement quasistatique alternatif est imposé à un solide élastique en contact avec un support rugueux, il a été mesuré expérimentalement que la force de friction décrit une boucle d'hystérésis. De plus, l'approche de la force de friction vers sa valeur asymptotique est exponentielle. Nous décrivons comment ce comportement est lié aux propriétés collectives des instabilités élastiques subies par le solide lorsqu'il est déplacé quasistatiquement. La longueur de relaxation de l'exponentielle est exprimée en terme de l'élasticité de la surface et des propriétés du support rugueux. Les lois d'échelle prédites sont confirmées numériquement.

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1 Introduction

The present paper presents a discussion of the hysteretic behaviour of the friction force between two solid bodies. We use a one dimensional model to develop a phenomenological analysis of friction in the quasi-static limit (zero velocity, *i.e.*, below the depinning threshold). In particular, we emphasize the connection between *macroscopic* friction and the *microscopic* elastic instabilities occurring when an elastic body is pulled over a microscopically rough surface. Guided by computer simulations of a one dimensional elastic chain, we construct a coherent collective pinning picture of friction. In particular, an equation for the relationship between the friction force and the number of elastic instabilities is derived. We use this equation to obtain the hysteretic relation between displacement and friction force. Our model is qualitatively in agreement with experiments on elastic friction [1].

1.1 Some open questions in friction

It is useful to divide the problem concerning the friction between two solid surfaces into a number of sub-problems. At the microscopic quantum level the forces have their origin in the electronic potential across the interface between the two surfaces. These forces can be measured by various types of scanning probe microscopy and can in some cases be calculated by use of a quantum theory for the surface atoms of the solids [2]. Microscopically the two surfaces may often be considered as ideal periodic structures and the interaction across the surfaces will be spatially homogeneous and periodic.

On the other hand, when two *macroscopic* surfaces are in contact, the forces between the surfaces will be due to the roughness of the surfaces on macroscopic length scales. The friction force will originate in the interlocking of a number of asperities on the two surfaces. If we can neglect plastic deformations (tear and wear) of the asperities as they are forced past each other, we may expect that it is possible to reduce part of the pinning problem to the interaction between macroscopic elastically deformable asperities. We can then neglect the particular quantum nature of the microscopic interaction and attempt to establish an effective phenomenological theory of the friction between macroscopic elastic surfaces. New controlled experiments on artificially constructed elastic surfaces make it urgent to understand theoretically this model system of friction [1]. Even this vastly reduced problem contains some rather subtle open questions.

Recently most of the theoretical and numerical works on the pinning problems have focused on the dynamical properties of friction in order to obtain

the friction-velocity relationship *above the depinning transition*, *i.e.*, when the applied force is greater than the critical pinning force [3, 4, 5]. In particular dynamical Renormalization Group techniques have been successfully applied in the low velocity limit by treating the depinning transition as a dynamical critical phenomena [4, 5].

On the other hand, the literature on the properties of pinning *below the depinning transition* is mainly restricted to the pioneering work of Larkin and Ovchinnikov (LO) in the 70's [7]. Their analysis first provided a sketch of the collective properties of pinning in order to construct the critical pinning force. Their predictions were successfully compared to experimental results on the pinning of vortex lines in superconductivity [7]. This success has led some authors to consider the behaviour below the critical threshold as essentially solved by LO [5]. However, in view of new experimental results on quasistatic friction and contact line motion [1, 8], it has become clear that the LO analysis is incomplete on the following important points. Firstly the LO argument does not involve *hysteretic behaviour*. Secondly their approach does not take any *mechanical instability* into account, while it has been shown that friction cannot arise without this crucial ingredient (see [9, 13] and the discussion below). Moreover, new experiments have shown the existence of *memory effects* in the approach to the critical pinning force, which are characterized by a length scale [1, 8, 10]. In the experiments of Heslot *et al.* [10], the introduced length accounts for the “age” of the pulled system, while in the experiments of Crassous *et al.* on friction and contact line motion [1, 8], this length characterizes the distance needed by the system to reach the stationary state. Therefore it is clear that although the LO analysis captures an essential part of the collective mechanism of pinning, the link to the above mentioned experimental observations is still lacking. The construction of this link is the main object of the present work.

1.2 The crucial role of elastic instabilities

As already pointed out by Tomlinson [11], friction arises in an elastic media due to the existence of a *multiplicity of metastable states*. This multiplicity induces mechanical instabilities in the system and leads to dissipation and hysteresis [9, 12, 13].

The instabilities arise whenever the local force-balance equation describing the interlocked surfaces becomes multivalued. As the mutual displacement of the two bodies is increased, the asperities are forced against each other. The interface force increases linearly as a function of the centre of mass displacement. When the local force balance becomes unable to sustain the mutual force between the

asperities, a local rearrangement of the atoms in the interface will take place. If the roughness of the interacting surfaces is sufficiently large, the motion of the interface atoms takes place in the form of swift jumps from one metastable configuration to another. During such an instability the friction force drops precipitously by a certain value, for then again to increase linearly upon further increase in the relative displacement of the two bodies.

Let us recall the microscopic features of the elastic instabilities. This is most easily done by use of a single degree of freedom picture[13, 9]. Consider a particle at position x elastically coupled to a position X (one may think of X as the center of mass of the elastic lattice). We want to follow the motion of the particle as it passes over an asperity modeled by a Gaussian peak in the potential energy. The energy of this system is given by

$$U = \frac{1}{2}\kappa(x - X)^2 + A_p \exp[-(x/R_p)^2]. \quad (1)$$

The static equilibrium of the system is obtained by solving the equation $\partial U/\partial x = 0$ for a prescribed X value, *i.e.*, we have to solve the equation

$$\kappa(x - X) = \frac{2A_p x}{R_p^2} \exp[-(x/R_p)^2]. \quad (2)$$

This equation is solved graphically in Fig. 6. The important point is that the equation has a single valued solution for any value of X as long as the slope of the force exerted by the Gaussian peak is everywhere smaller than κ . This condition is

$$\frac{2A_p}{R_p^2} < \kappa. \quad (3)$$

When this condition is not fulfilled (as in Fig. 6), the equilibrium position of the particle $x(X)$ will become a discontinuous function of X : a mechanical instability occurs when the system passes over the defect. Accordingly, energy is dissipated (e.g. into the rapid degrees of freedom, like phonons) : instabilities induce hysteretic behaviour. However, this single particle mechanism does not lead to memory effects. Therefore, in a more general case where an elastic medium slides over many defects, one still expects the basic mechanism for friction to be linked to the occurrence of elastic instabilities; on the other hand, memory effects are the indication of the collective character of instabilities.

1.3 Our aims

Our aim in this paper is to give a mainly qualitative picture of collective pinning in the quasi-static limit in order to understand the memory effects. We stress

the fact that as in the experiments of ref. [1], we study the system in a regime *below the depinning transition*: the sliding velocity of the two solid bodies strictly vanishes and no dynamical effects are expected in our case. In particular the system will be considered at equilibrium under an applied constraint. The velocity dependence of the friction force is therefore not the purpose of the present study [3, 4, 5].

We will attack the problem along the following line :

(i) First we perform numerical simulations of a very simple model of the system, in order to check if the latter is able to reproduce the experimental results of ref. [1]. In particular, the following experimental facts should be recovered: existence of an **hysteresis loop** when the system is pulled back and forth over the rough surface; existence of a **memory length**; **exponential approach** towards the stationary friction force.

(ii) Then we construct a “microscopic” scenario of friction by analysing the microscopic behaviour of the system.

(iii) Finally, on the basis of the numerical results, we propose a qualitative phenomenological model for the collective properties of friction in the quasi-static limit. This model provides the link between the collective *equilibrium* and *out-of-equilibrium* properties of the system below the depinning threshold.

2 General features of the numerical results

2.1 A simple numerical model

Let us first introduce the model we use for our numerical experiments. For simplicity we model the two interacting elastic surfaces by a mobile deformable elastic medium in contact with a stationary undeformable rough surface. This is clearly a limitation in comparison to two deformable elastic surfaces. However, our findings will justify our anticipation that this simplification is inessential.

Our model consists of a one dimensional string of particles at positions x_i where $i = 1, \dots, N$, coupled together through elastic springs all of the same spring constant k . The chain, of length L , is assumed to be periodic. We use the equilibrium length a of the elastic springs as our unit of length $a = 1$. The particles interact with a set of pinning centres in the form of randomly positioned (at positions x_i^p where $i = 1, \dots, N_p$) repulsive asperities of density n_p . All the asperities are modelled by the same Gaussian potential peak of amplitude A_p and

range R_p . The potential energy of the system can accordingly be written as

$$\begin{aligned}
U &= U_{el} + U_{pin} \\
&= \frac{k}{2} \sum_{i=1}^{L-1} (x_i - x_{i+1} - a)^2 + \\
&\quad \sum_{i=1}^L \sum_{j=1}^{N_p} A_p \exp[-(x_i - x_j^p)^2 / R_p^2].
\end{aligned} \tag{4}$$

We are interested in experiments where dynamical effects can be neglected. For this reason we investigate the total force produced by the asperities when the elastic chain is moved *quasistatically* through the asperities.

The simulations are performed in the following way. We start from the ideal lattice configuration of the elastic chain. We then use molecular dynamics annealing to relax the chain to the substrate potential. This is done by starting the particles out with a random distribution of velocities. The Newtonian equation of motion derived from the potential in Eq. (4) is then integrated by the leap-frog algorithm [13]. While integrating the equation of motion we gradually extract kinetic energy by scaling the velocities of the particles. In this way we move the system towards one of the metastable configurations of low potential energy.

When this initial relaxed configuration has been prepared we study the friction (or “pinning”) force induced when the centre of mass (c.o.m.) of the chain is gradually forced through the asperities at zero velocity $V \equiv 0$. The most accurate way of numerically simulating the quasistatic displacement of the c.o.m., X , is by performing cycles of shift and relaxation. The elastic chain is displaced as a *rigid* body a small amount dx ($dx/a = 10^{-5}$ in our simulations) by replacing all the particle positions by $x_i \mapsto x_i + dx$. While the c.o.m. is kept fixed at this new c.o.m. position $X_{new} = X_{old} + dx$, the chain is next relaxed to the asperities. The c.o.m. is kept fixed by always counteracting the force exerted by the substrate by an external force f_{ext} applied homogeneously to all the particles of the chain. We have

$$f_{ext} = \frac{1}{L} \sum_{i=1}^L \frac{\partial U_{pin}}{\partial x_i} \tag{5}$$

where f_{ext} used in the present time step is calculated from the positions of the previous molecular dynamics time step. This method allows one to follow as accurately as desired the motion through the background potential. One could of course also move the chain by applying an external force larger than the present friction force. This method leads to results qualitatively equivalent to the above described method. The draw back of simply applying a constant external force is that the force from the substrate fluctuates in space. A constant force will therefore sometimes be much larger than the pinning force. This leads to unwanted

acceleration effects and make it difficult to remain in the limit of quasistatic motion [15].

2.2 Existence of hysteresis

As in the experiments of ref. [1], we move the elastic chain back and forth over the “rough” surface, according to the numerical algorithm discussed above. For each c.o.m. position of the chain, $X_{c.o.m.}$, we measure the force acting on the particles of the elastic chain. This force will be denoted as the friction force, F_f . The whole curve F_f vs. $X_{c.o.m.}$ is then averaged over many cycles and over different random spatial configurations of pinning centres. Fig. 6 shows a typical numerical result. Starting from a given point, the friction force reaches *after a finite distance* a plateau value, independent of the c.o.m. position. This plateau value is the **static friction force** : it is the maximum value for an external force before the solid body (here, the elastic chain) acquires a non vanishing velocity. Then, when the system is moved in the other direction, the same plateau value with the opposite sign is reached, but following a different curve in the F_f versus $X_{c.o.m.}$ plane : an hysteresis loop is performed during a cycle.

As in the experiments, memory effects are clearly observed, since it takes a finite *length* for the system to reach the stationary pinning force. This defines a memory length.

2.3 Exponential decay of the pinning force

As shown in Fig. 6, the approach towards the plateau value for the friction force is exponential. For example, when the system is pulled in the $X_{c.o.m.} > 0$ direction, the friction force can be very well fitted after an initial small transient distance, by the following relation :

$$F_f(X_{c.o.m.}) = F_\infty + (F_0 - F_\infty) \exp \left\{ -\frac{X_{c.o.m.} - X_0}{\zeta} \right\} \quad (6)$$

where F_∞ is the plateau value for the force, F_0 is the force measured at a given point X_0 . This relation defines the memory length, ζ . In our simulation, the latter was obtained to be of the order of a few R_p , the range of the defects. The size of the initial transient regime was obtained to decrease when the density of pinning centres n_p increases.

This exponential approach towards the plateau value of the pinning force is in perfect agreement with the experimental results of ref. [1]. Therefore, the simple

numerical model we use should contain the essential of the underlying physics leading to the memory effects and exponential decay. We can now analyse in greater details the simulation results to give a microscopic picture leading to these results.

3 A first empirical understanding

3.1 Hooke’s law and instabilities

While in Fig. 6, we considered an average of the friction force over many different initial states, we now focus our attention on a particular realization of the numerical experiment. A typical non-averaged plot of the friction force as a function of the c.o.m. displacement is shown in Fig. 6. This figure is characterized by a saw-tooth behaviour, which splits up into *linear increase* of the friction force, separated by *steep decrease* in the friction force.

(i) The linear behaviour corresponds to the **reversible** linear (elastic) response of the system, when an external constraint is applied in order to impose a given c.o.m. displacement. This response is characterized by an elastic susceptibility, κ_L , defined by

$$dF_f = \kappa_L \cdot dX_{c.o.m.} \quad (7)$$

relating the measured infinitesimal change in the friction force, dF_f , to the c.o.m. displacement $dX_{c.o.m.}$, according to a simple Hooke’s law. The parameter κ_L was first introduced by Labusch [16], in the context of lattice deformations in crystals and will be denoted as the “Labusch parameter” in the following.

(ii) The discontinuities in the force are the indication of a dramatic **irreversible** transformation occurring in the system. A look at the microscopic trajectories shows that these jumps in the force are intimately connected to a large *common* displacement of a significant number of particles (typically of order 10-50 over 500). The typical displacement δx_0 of each particle of the moving block was always of the order of one lattice spacing in all our simulations : $\delta x_0 \simeq a$. This indicates that, because of the imposed c.o.m. displacement, the local equilibrium state of the system becomes unstable and a finite jump occurs towards a new local equilibrium state [13]. In other words, these jumps occur because the system cannot support elastically the applied external constraint anymore, so that a new metastable state has to be found in order to release the stored energy. Dissipation occurs during these abrupt transitions. In the following, these jumps will be denoted as “elastic instabilities”.

The previous scenario is in fact very reminiscent of the single particle case (*i.e.*, one spring over one defect), discussed above [9, 17]. In particular, the multistability of the metastable positions seems thus to be recovered in the many-particle system. However the characterization of the associated instabilities is now much more difficult, because of the collective character of the phenomena, as will be shown in section 4.

Let us assume that the elastic chain is pulled in the $X_{c.o.m.} > 0$ direction. We introduce the spatial frequency of instabilities, ν , defined as the number of instabilities occurring in the system per unit c.o.m. displacement; and ΔF_{inst} the drop in force occurring in a single instability. It is useful to *define* it to be the absolute value of the drop, so that $\Delta F_{inst} > 0$. The latter is assumed, of course, to depend on the c.o.m. position (or equivalently on the friction force) at which the instability takes place. According to the previous scenario, a simple differential equation for the (averaged) friction force, F_f , can be written :

$$\frac{dF_f}{dX_{c.o.m.}} = \kappa_L - \nu \langle \Delta F_{inst} \rangle \quad (8)$$

which accounts for both the elastic response of the system and the finite change in the force during instabilities. The notation $\langle \dots \rangle$ means a non-equilibrium average over many different distributions of pinning centres *for a given c.o.m. displacement*, or equivalently *for a given friction force*. We omit the brackets for the Labusch parameter κ_L , since the latter is found in the simulations to be independent of the friction force F_f . This means that the elastic susceptibility of the system remains constant, when the friction force increases, *i.e.*, when new metastable states are explored. This rather astonishing fact will be discussed briefly in section 5.

We can see in Fig. 6 that no instability occurs until the force has reached the value $F_f = 0$ and a rather long elastic relaxation takes place up to that point : then $\nu = 0$ for $F_f < 0$. In fact, this is to be expected since in our numerical procedure (involving an alternative forward/backward displacement of the whole system like in the real experiments), the initial state is *not* an equilibrium state but the stationary state of the system when it is pulled in the opposite direction. Therefore, until some stress is *effectively* supported by the system (*i.e.*, $F_f > 0$), the system relax elastically the external constraint and no instability occurs. For $F_f > 0$, the instability process is turned on. The frequency of instabilities is found numerically to reach a plateau value in a very short distance (much smaller than the lattice spacing, a), followed by a small decrease towards its stationary value. This small F_f dependence will be omitted in the following and the frequency will be considered as roughly constant (for $F_f > 0$).

3.2 A phenomenological law and the exponential decay

The averaged change in force during an instability, $\langle \Delta F_{inst} \rangle$ does not remain constant when the c.o.m. is displaced. This can be seen for example in Fig 6, where the non-averaged friction force is plotted versus the c.o.m. displacement. The change in force during an instability is seen to increase when the system is pulled quasi-statically, so that $\langle \Delta F_{inst} \rangle$ is expected to increase when the friction force F_f increases. We conjecture a simple **linear relationship** between these two quantities :

$$\langle \Delta F_{inst} \rangle = \delta F_0 + \alpha F_f \quad (9)$$

where δF_0 and α are two phenomenological parameters

This “phenomenological law” has been checked in the simulations by plotting the averaged change in an instability as a function of the friction force measured just before the instability takes place. Numerically, this procedure involves a simple algorithm which detects the instabilities. The latter relies on the measure of the numerical derivative of the friction force, which exhibits a dramatic change during an instability. This rough indicator has been checked to work with a very good accuracy. We have then averaged the plot over many different realizations of the initial conditions to compute *for a given friction force*, the corresponding averaged change in force $\langle \Delta F_{inst} \rangle$. A typical result is plotted in Fig. 6. Except in the large force region, the numerical points can be fitted with a good agreement by a straight line, thus validating the phenomenological relation in eq. (9). Physically, the increase of the force release during an instability is understandable. Roughly, when the external constraint increases, the “susceptibility” of the system increases accordingly, so that the drop in force during an instability becomes larger. This point will however be studied in more details in the next section. The increase of $\langle \Delta F_{inst} \rangle$ for large forces in Fig. 6 could be the indication of the onset of large fluctuations arising in the close vicinity of the depinning threshold, although “critical” avalanches (*i.e.*, involving the whole system) were not observed in our simulations. But this problem requires a specific careful numerical work as done by Pla and Nori [14]. This is not the object of the present work.

Combining the phenomenological relation eq. (9) with eq. (8) leads to a *closed equation* for the friction force :

$$\begin{aligned} \frac{dF_f}{dX_{c.o.m.}} &= \kappa_L - \nu \langle \Delta F_{inst} \rangle \\ &= (\kappa_L - \nu \delta F_0) - \nu \alpha F_f \end{aligned} \quad (10)$$

This equation predicts an **exponential relaxation** of the friction force towards

a stationary value given by

$$F_f^\infty = \frac{\kappa_L}{\nu\alpha} - \frac{\delta F_0}{\alpha} \quad (11)$$

The relaxation length, ζ , is related to the phenomenological parameter α , through the simple relation :

$$\zeta = 1/\nu\alpha \quad (12)$$

This exponential behaviour is in agreement with both the numerical results (see Fig. 6) and the experimental results of Crassous *et al.* [1].

Therefore, the phenomenological relation eq. (9) provides an initial “empirical” understanding of the underlying physics of friction. However, how this simple law emerges from the microscopic picture remains to be clarified. Moreover, the results obtained within this approach for the stationary force (eq. (11)) and relaxation length (eq. (12)) are only useful, if some prediction can be made for their dependence on the physical parameters of the system (e.g. density of pinning centres n_p , strength and range of the pinning centres, A_p, R_p). This is the object of the next section.

4 Microscopic picture : towards collective pinning

4.1 Description of instabilities

In paragraph 3.1 we described the instabilities as an instantaneous collective motion of an important number of particles of the system, while the other remained more or less stationary. Intuitively, it would be appealing to relate the drop in the friction force to the number of jumping particles during the instability. To this end, we have studied numerically the microscopic behaviour of the system when an instability takes place. Especially, after isolating an instability, we separated the system into the block of “jumping particles” and the remaining particles. For both parts of the system, we computed the change in the force due to the defects. This operation was repeated for a number of instabilities of different amplitudes, taking place at different c.o.m. positions. The general “rule” we obtained was the following : (i) the change in force measured for the jumping particles only was found to be *very small*; (ii) the global drop in the total friction force observed in an instability is caused by the remaining particles, which didn’t undergo a large displacement. This can be interpreted as follows. When each of the N_{jump} jumping particles move *forward* by a large, finite amount δx_0 , the c.o.m. of the

remaining particles will move *backward* by an amount $\Delta x_{c.o.m.} \simeq -N_{jump} \delta x_0 / N$, in order to keep the c.o.m. of the complete system constant. The corresponding drop in force is then simply given by the relaxation of the corresponding elastic constraint, according to Hooke's law (see eq. (7)) :

$$\begin{aligned} F_{after} - F_{before} &\equiv -\Delta F_{inst} &= \kappa_L \Delta x_{c.o.m.} \\ &= -\frac{\kappa_L \delta x_0}{N} N_{jump} \end{aligned} \quad (13)$$

Eq. (13) provides the desired link between the drop in the force occurring in an instability and the corresponding “length” of the instability, *i.e.*, the number of particles that jumped, N_{jump} .

4.2 Existence of a coherence length

The afore-mentioned point (i) - that the change in force measured for the jumping particles only is extremely small - is very striking. In fact, this can be considered as a first indication of the *collective character of the instabilities*. To understand this subtle point, we first recall how an instability occurs in the single particle case, as discussed by Nozières and Caroli [9, 13]. The system involves then only a single particle attached to a spring and interacting with a defect. The extremity of the spring is moved adiabatically. For a sufficiently soft spring (see section 1.2 for details), the equilibrium position of the particle becomes multi-valued over a given range of system positions, *i.e.*, the system becomes unstable. The evolution of the particle thus exhibits three phases when it encounters the defect : first an adiabatic move, corresponding to the elastic response of the defect; then a large finite jump towards a point where the reaction force is very small : this corresponds to the elastic “instability”; finally, an adiabatic move again, where only the tail of the defect's potential is felt by the particle. Accordingly, the drop in the force Δf during the instability is of the order of the threshold force before the jump. Now consider N_{jump} particles jumping according to such an “individual scenario”, the global change in force for the whole system would simply be $\Delta F_{inst} \simeq N_{jump} \Delta f$, which is proportional to the number of particles involved in the instability. This assumes that the drop in the force originates from the particles involved in the instability. But **this differs dramatically** from our observations in the numerical experiments. Indeed according to the previous points (i) and (ii), the crucial difference is that in our simulated systems, the drop in the force during an instability is only induced by the release of the elastic response of the particles *not involved* in the instability.

We interpret this point by assuming that the system separates into two types of particles : a few particles interact “strongly” with the defects, while all the other only interact weakly with the defects, *i.e.*, they can be considered as

“strongly correlated”. The equilibrium positions of one (or a few) strongly pinned particle may become unstable, the latter thus performing a large finite jump. All particles “strongly connected” to this particle then simply follow collectively the jump of the first jumping particle. Within this scenario, the change in force due to the “jumping part” of the system has no reason to be large. Indeed, most of the jumping particles (*i.e.*, except for the one -or the few- strongly “pinned”) do not interact strongly with the defects, so that their individual change in force during their jump is mostly random, justifying the observation facts (i) and (ii).

There is another indication of the “collective” character of the instabilities. Let us come back to the phenomenological relation (9) for the drop in the force during an instability, verified numerically on Fig. 6. As can be observed in this figure, the value at the origin, $\delta F_0 = \Delta F_{inst}(F_f = 0)$, is non-vanishing. This means that, as soon as instabilities are allowed, a finite (non-zero) number of particles are involved in the instability. Therefore, the length scale of the previously discussed “strongly connected” particles is non vanishing at equilibrium, and is hence an intrinsic property of the system at equilibrium.

These observations lead us to the notion of “strongly pinned” particles and “strongly connected” particles, the latter being characterized by a well defined correlation length. In fact, this separation is reminiscent of the analysis introduced by Larkin and Ovchinnikov (LO) [6, 7], of the pinning of vortex lines in superconductors. Their analysis is based on the introduction of a “correlation length”, characterizing the balance between the elastic energy (of the Abrikosov lattice in superconductors) and the pinning energy due to the interaction with the defects. This length can be equivalently interpreted as the displacement correlation length of the system. More precisely, the linear size of the correlated volume l_L is defined as the length scale over which the elastic rigidity of the elastic medium is sufficient to counteract the random forces. The relative shear induced by the random forces is small within a correlated volume. The particles within a correlated volume are strongly correlated in the sense that if a particle is displaced a small amount, this displacement will be transmitted by the rigidity of the lattice out to distances of the order of the size of the correlated volume. At distances larger than the size of the correlated volume the random forces become essential. The distortion of the elastic lattice becomes appreciable and particles farther apart than l_L are only weakly correlated. In this sense we can think of the elastic lattice as being broken into weakly interacting rigid subvolumes. It is at the interface between the correlated volumes that the elastic instabilities occur.

We now recall the (qualitative) LO argument in order to estimate ℓ_L .

In the picture depicted above, the total pinning force on a correlated volume, *i.e.*, the force due to the defects, is the sum of many contributions of the same

order $f_0 \sim A_p/R_p$, but with *different signs* (since the strongly correlated particles only interact weakly with the pinning centres). Therefore, the pinning force on a correlated volume involves a statistical summation over all the individual contributions and will be of order $N_c^{1/2} f_0$, where $N_c = n_p \ell_L$ is the number of pinning centres included in the correlated volume (we now restrict our study to a 1-D problem). Since there are L/ℓ_L correlated volumes in the system, the total pinning force is found to be of order :

$$F_{pin} \sim L \left(\frac{n_p}{\ell_L} \right)^{1/2} f_0 \quad (14)$$

Since this force only acts for a distance of order R_p (the range of the pinning potential), before changing randomly, the pinning energy is assumed to behave like

$$\delta \mathcal{E}_{pin} \sim L \left(\frac{n_p}{\ell_L} \right)^{1/2} f_0 \cdot R_p \quad (15)$$

On the other hand, the increase of elastic energy due to the deformation of the lattice inside a correlated volume can be estimated as $\delta \mathcal{E}_{el} = \frac{1}{2}(k/n\ell_L) \delta x^2$, where k is the bare spring constant, $n = 1/a$ is the density of particles (a is the spring length), and δx is the distortion distance. The term $k/n\ell_L$ takes into account the *effective* stiffness of $n\ell_L$ springs contained in a correlated volume. The distortion length is expected to be of order of R_p , the range of the defect potential. For the whole system (involving L/ℓ_L correlated volumes), we thus find

$$\delta \mathcal{E}_{el} \sim L \frac{1}{2} k a \left(\frac{R_p}{\ell_L} \right)^2 \quad (16)$$

Adding eqs. (15) and (16), we find the change in energy per unit length associated with the randomly distributed defects :

$$\delta \mathcal{F} = \frac{1}{2} k a \left(\frac{R_p}{\ell_L} \right)^2 + \left(\frac{n_p}{\ell_L} \right)^{1/2} f_0 \cdot R_p \quad (17)$$

The optimized correlation length is found by assuming that the system evolves towards a state which minimizes the costs in energy. This is done by minimizing the expression (17) with respect to ℓ_L , leading to

$$\ell_L = \left[\frac{2k a R_p}{n_p^{1/2} f_0} \right]^{2/3} \quad (18)$$

Replacing (18) into eq. (14), we obtain a total pinning force which scales like

$$F_{pin} \sim L \cdot \frac{n_p^{2/3} f_0^{4/3}}{k a R_p^{1/3}} \quad (19)$$

Larkin and Ovchinnikov [7] assumed that the pinning force in eq. (19) is **the out-of-equilibrium pinning force**, defined as the force measured at the point where the systems depins, *i.e.*, acquires a non-vanishing velocity. The latter corresponds to the asymptotic force measured in our numerical experiments (or in the *real* experiments of ref. [1]) when the system is pulled quasi-statically.

But our interpretation of F_{pin} differs qualitatively from the previous affirmation. Indeed, the LO argument as depicted here is an **equilibrium** argument, which allows one to find the equilibrium properties of a lattice interacting with randomly distributed defects. In this case, the averaged pinning force should vanish (otherwise the system is not at equilibrium). Therefore the “pinning force” defined in eq. (14) cannot be identified with the out-of-equilibrium pinning force. Moreover, the LO argument does not take the elastic instabilities into account, while the numerical simulations (and even the real experiments, see [18] and [19]) show that these instabilities play a crucial role in the pinning process.

In contrast to Larkin and Ovchinnikov, we interpret F_{pin} of eq. (14), obtained along the previous argument, as a typical underlying scale for the *fluctuations* of the reaction force due to the defects at equilibrium : more precisely, it fixes the amplitude of the root-mean-square (r.m.s.) fluctuations of the pinning force. How this scale is related to the non-vanishing, non-equilibrium pinning force (*i.e.*, the plateau value for the force for example in Fig. 6), remains to be understood and will be the object of the next sections.

4.3 Linking the coherence length and the phenomenological relation

In order to make the connection between the equilibrium properties of the system and our empirical understanding of friction depicted in section 3, we make the following two **conjectures** :

(A) in an instability, the number of particle undergoing a jump is given by the length of the correlated volume. In other words, the instability is defined by a block-jump of an entire correlated volume.

(B) when an external constraint is applied, the length of the correlated volume increases.

Let us first discuss these two assumptions. The first one, (A), is intuitively understandable. Indeed, according to the picture already discussed in the previous paragraph, instabilities are initiated by the few particles which are strongly interacting with the defects. When one of these strongly interacting particles

becomes unstable, and thus performs a large jump, all the particles which are strongly “attached” to this jumping particle will follow as a whole.

The second assumption, (B), can be justified in the following way. The correlation length, ℓ_L depends quite strongly on the strength A_p of the pinning centres, as can be seen in eq. (18) (remember that $f_0 \sim Ap/R_p$): the weaker the pinning centres are, the longer the correlation length is, because the elastic lattice is then less distorted. Now, when an external force is applied, the *effective* strength of the defects will decrease, so that the correlation length is expected to increase when the system is moved, justifying conjecture (B). To lowest order in the external force, we may expect a *linear* increase of the correlation length, as would be obtained by applying linear response theory to the system :

$$\ell_L(F_{ext}) = \ell_L^{equ} + \gamma \cdot F_{ext} \quad (20)$$

where ℓ_L^{equ} is the equilibrium correlation length obtained along the LO argument, defined in (18). F_{ext} is the total external force applied to the system in the direction of the motion : $F_{ext} = \mathbf{F}_{ext} \cdot \hat{\mathbf{e}}$, with $\hat{\mathbf{e}}$ the unit vector pointing the direction of motion. In our case, the external force applied in order to impose a given c.o.m. position simply identifies with the measured friction force, F_f , so that $F_f = F_{ext}$. Note that, at first sight, considerations of symmetry would only predict a F_{ext}^2 dependence of ℓ_L as a function of the external field (because of the symmetry $F_{ext} \rightarrow -F_{ext}$). However such a counter-intuitive linear behaviour usually occur in degenerate systems, where a careful perturbation theory has to be done [20]. This is illustrated for example on the so called “Stark effect” for hydrogen, where a first order change in the energy of the first excited state (which is fourfold degenerated) is found when an electric field is applied. In our case, the degeneracy stems from the multistability of the metastable states, which defines spatial “multipoles”. The linear change in ℓ_L may be thought as arising from the interaction between these multipoles and the applied field. Moreover, the linear guess will be confirmed *a posteriori* by the consistency of the scenario which results from it.

By use of the two conjectures, we can now understand the “phenomenological relation”, eq. (9), relating linearly the drop in force measured an instability to the friction force. Combining eqs. (20) and (13), and setting $N_{jump} = n \ell_L(F_f)$ according to the previous argument, we obtain

$$\begin{aligned} \Delta F_{inst} &= \frac{\kappa_L n \ell_L(F_f) \delta x_0}{N} \\ &= \frac{\kappa_L n \ell_L^{equ} \delta x_0}{N} + \left(\frac{\kappa_L n \gamma \delta x_0}{N} \right) \cdot F_f \end{aligned} \quad (21)$$

The proposed scenario thus gives a coherent picture of the mechanism leading to the phenomenological law. Moreover, identifying this expression with the phenomenological relation eq. (9), we are left with the following “microscopic” expressions for the two phenomenological parameters δF_0 and α :

$$\begin{aligned}\delta F_0 &= \frac{\kappa_L n \ell_L^{equ} \delta x_0}{N} \\ \alpha &= \frac{\kappa_L n \gamma \delta x_0}{N}\end{aligned}\tag{22}$$

One of the important consequences of eqs. (22) is that it will allow us to compute the dependence of the two introduced phenomenological quantities on the microscopic parameters of the system (density of pinning centres n_p , strength and range of the defects A_p , R_p). This requires more information on the Labusch parameter κ_L , which will be obtained in the next section.

For the dimensionless parameter α , the derivation is more subtle since it involves the new parameter γ introduced in eq. (20). If we think of this parameter in terms of a susceptibility (as usually done in linear response theory), then it is determined by the properties of the system *at equilibrium* [21]. Now, according to its definition (20), it has the dimensionality of a length divided by a force. But for a system of springs interacting with randomly distributed defects, we expect the length scale to be fixed by the correlation length scale introduced in the LO argument (see eq. (18)), while the force should be fixed by a typical value of the r.m.s. fluctuations of the pinning force. According to our interpretation of the LO argument, this scale is fixed by F_{pin} , introduced in eq. (14). Therefore we are left with the following expression for the parameter γ :

$$\gamma \sim \frac{\ell_L^{equ}}{F_{pin}} \sim \ell_L^{equ} \left(\frac{\ell_L^{equ}}{n_p} \right)^{1/2} \frac{1}{L f_0}\tag{23}$$

By replacing eq. (23) into eq. (22), one obtains for the phenomenological parameter α the expression

$$\alpha \sim \frac{\kappa_L}{N} n \delta x_0 \cdot \frac{\ell_L^{3/2}}{L f_0 n_p^{1/2}}\tag{24}$$

where the expression for ℓ_L is given in eq. (18). This expression can be written in a more transparent form :

$$\alpha \sim \frac{\delta F_0}{F_{pin}}\tag{25}$$

The predictions of eqs. (22) and (24) will be compared with the numerical results in section 5.3. This will allow us to assess the validity of our approach. But we first need to characterize more properly the dependence of the other quantities, in particular of the Labusch parameter. This is done now.

5 Friction and scaling laws

5.1 The Labusch parameter

The Labusch parameter is defined in eq. (7) as the elastic susceptibility of the system. By definition, this susceptibility measures the response of the pinning centres (defects) when the particles interacting through the springs are slightly displaced from their equilibrium positions, *i.e.*, when the applied force balances the pinning force [15]. We performed numerically this “experiment” by displacing the c.o.m. by a very small amount and analysed the corresponding change in force on each defect. Surprisingly, the change in the reaction force of the defects was found to be homogeneously distributed over the defects, thus leading to a picture of an individual elastic response of the pinning centres to the displacement of the spring system.

Therefore, we expect the Labusch parameter to scale like

$$\kappa_L = n_p L k_0 \quad (26)$$

where n_p is the density of pinning centres and L the length of the system. k_0 is the individual elastic response of a defect, and can be roughly estimated as the probability for a defect to interact with a particle, of order $\sim n R_p$ (n being the particle density and R_p the range of the potential), multiplied by a typical value of the second derivative of the pinning potential, of order A_p/R_p^2 . This leads to

$$k_0 \sim n R_p \frac{A_p}{R_p^2} \quad (27)$$

Combining, eqs. (26) and (27), we obtain that the Labusch parameter should scale like

$$\kappa_L \sim n_p A_p R_p^{-1} \quad (28)$$

This scaling relation has been checked in the simulations, by varying n_p for a given set of potential parameters A_p , R_p , and varying A_p , R_p for a given density. The numerical results are obtained to be in reasonable agreement with these predictions. The dependence on n_p is shown on Fig. 6 : the numerical value of κ_L is plotted as triangles; the underlying dotted curve is a straight line with slope 1. We explored numerically the dependence on the strength and range of the defect potential too. This was done by varying slowly A_p and R_p for a given density. The presented results do not involve however an average over different random spatial distributions of the pinning centres. The measured points for κ_L are plotted as circles in Fig. 6 for the dependence on A_p and in Fig. 6 for the dependence on R_p : The dotted lines are a guide for the eye to indicate the

predicted slopes : slope 1 for the dependence on A_p (in Fig. 6) and slope -1 for the dependence on R_p (in Fig. 6). The trend is seen to be correct for both dependences. However a more extensive numerical study is still needed for a whole set of parameters A_p and R_p , using different densities of pinning centres.

It is interesting to note that the picture of an individual response of the pinning centres for the elastic susceptibility of the system is coherent with the fact that the Labusch parameter is found in the simulations to be independent of the friction force (*i.e.*, of the applied constraint). We confirmed this result in all our simulations, by checking that the slope of the linear part of the non-average friction force, as in Fig. 6, do not change when the c.o.m. of the system is displaced. Apart from the previous qualitative argument, based on an individual response of the pinning centres, this result remains up to now quite obscure to us.

5.2 The Frequency of instabilities

Finally, to complete our phenomenological description, we need to characterize more precisely the *frequency* of instabilities, ν . Up to now we do not have a full understanding of the mechanism of *creation* of instabilities, but some predictions can however be made.

In the *stationary* regime, the following relation is obtained from eqs. (8) and (21)

$$\kappa_L = \nu \Delta F_{inst} = \nu \frac{\kappa_L n \ell_L(F_f^\infty) \delta x_0}{N} \quad (29)$$

where F_f^∞ is the plateau value of the friction force. This equality imposes the scaling of the asymptotic frequency ν to be

$$\nu \sim \frac{N}{n \delta x_0} \frac{1}{\ell_L^{equ}} \quad (30)$$

Note that here we are only interested in the *scaling* properties of the frequency ν , so that we forget any friction force dependence of ℓ_L to focus on its dependence on density of defects and other microscopic parameters : we simply use the fact that the stationary value of the correlation length $\ell_L(F_f^\infty)$ has the same scaling on n_p , etc... as ℓ_L^{equ} defined in eq. (18). We cannot a priori apply the expression for ν in eq. (30) to the transient regime.

However an alternative argument can be given, which estimates ν from the fact that the system is broken up into the weakly and strongly pinned particles. Recall the mechanism producing the instabilities in the single particle problem

(see 1.2.) : a particle P (position x) is elastically coupled to a position X and interact with a defect. In the elastic region, we may approximate the force due to the defect by a Hooke's law, with a stiffness $k_d \sim A_p/R_p^2$. Then, if the position X is displaced by a small amount δX , the equilibrium position of the particle P will be displaced by an distance $\delta x \simeq \kappa \delta X / (\kappa + k_d)$, where κ is the spring constant. If the stiffness of the spring is much weaker than k_d (which is a condition for the existence of instabilities, see e.g. (3)), we obtain $\delta x \sim \kappa \delta X / k_d$. Now in our many body system, we can identify the position X with the c.o.m. position $X_{c.o.m.}$, and the stiffness of the spring κ with the effective stiffness of the correlated volume, $\kappa \sim k / (n \ell_L)$. Moreover the particle P can be identified with one strongly pinned particle, since in our microscopic scenario the latter are expected to induce the instabilities. An instability will occur when the latter will move over a distance of order of the range of the potential : $\delta x \sim R_p$. This is equivalent to a change in c.o.m. position δX given by $\delta X \sim n \ell_L k_d R_p / k$, according to our single particle discussion. This argument thus predicts a frequency scaling like

$$\nu \sim \frac{k}{k_d n R_p} \frac{1}{\ell_L^{equ}} \quad (31)$$

which is consistent -though not strictly identical- with our first guess, eq. (30).

In particular, this argument predicts a dependence of the frequency of instabilities on the density of pinning centres as $\nu \sim n_p^{1/3}$ (see eq. (18)). The numerical results are consistent with this scaling as shown in Fig. 6 : the circles are the numerical points; the dotted line has a slope 1/3.

5.3 Summary of the predictions

At this stage, we can check the predictions for the phenomenological parameters and measured quantities obtained within our scenario : in particular for the two phenomenological parameters δF_0 and α , for the Labsuch parameter, the plateau value of the friction force F_f^∞ and the memory length ζ . Apart from the pinning force and the Labusch parameter, we have mainly focussed our numerical study on the dependence of the measured quantities on the density of pinning centres n_p . Since the predicted scalings on n_p are not obvious (see the different powers of n_p in the equations below !), the comparison with the numerical results as a function of n_p are therefore expected to be already a drastic test for our scenario. But clearly more extensive numerical work needs to be done to explore the whole parameter space (n_p, A_p, R_p).

First we focus on the phenomenological parameters, δF_0 and α . Their “microscopic” expressions are given in eqs. (22) and (24). Combining the predicted

scaling of the Labusch parameter, as displayed in eq. (26) with the expression of the correlation length ℓ_L^{equ} , given in eq. (18), we obtain the following dependence as a function of the density of pinning centres, n_p :

$$\begin{aligned}\delta F_0 &\sim n_p^{2/3} \\ \alpha &\sim n_p^0\end{aligned}\tag{32}$$

In particular, this shows that the slope α of the phenomenological law eq. (9) does not depend on the density of pinning centres ! Both predictions of eq. (32) have been checked numerically. The results are plotted in Fig. 6 : symbols represent the numerical points and the dashed lines, the predicted scalings. The agreement is seen to be correct for both parameters. This confirmation is crucial, since it shows that the proposed scenario for the underlying mechanism of friction (see section 4) is coherent. In particular, this justifies our two conjectures (A) and (B) of section 4.3.

Now we turn our attention to the other measured quantities. According to eq. (11), the stationary friction force F_f^∞ is the sum of two term, $\kappa_L/\nu\alpha$ and $\delta F_0/\alpha$. However combining the “microscopic” expressions obtained for all the different parameters involved, eqs. (22),(24),(25),(30), we obtain that **the stationary friction force F_f^∞ simply scales like F_{pin} , the r.m.s. value of the fluctuations in equilibrium**. Using eq. (19), this leads to

$$\begin{aligned}F_f^\infty &\sim F_{pin} \sim L \frac{n_p^{2/3} f_0^{4/3}}{k a R_p^{1/3}} \\ &\sim n_p^{2/3} A_p^{4/3} R_p^{-5/3}\end{aligned}\tag{33}$$

On the other hand, using (24),(25),(30), the memory length ζ is found to scale like

$$\begin{aligned}\zeta &= \frac{1}{\nu\alpha} \\ &\sim \frac{F_{pin}}{\kappa_L} \sim n_p^{-1/3} A_p^{1/3} R_p^{-2/3}\end{aligned}\tag{34}$$

where the microscopic expressions for F_{pin} and κ_L have been used (see eqs. (19),(26)).

In fact, an equivalent relaxation length can be introduced, defined as the ratio between the asymptotic friction force F_f^∞ and the Labusch constant κ_L :

$$\delta \equiv \frac{F_f^\infty}{\kappa_L}\tag{35}$$

Since F_f^∞ scales like F_{pin} , this length exhibits the *same scaling* as the memory length ζ :

$$\delta \sim \zeta \sim n_p^{-1/3} A_p^{1/3} R_p^{-2/3} \quad (36)$$

On Figs. 6 and 6, we summarize the numerical results for the scalings of the different quantities as a function of the density of pinning centres. The symbols are the numerical points and the dashed lines indicate the predicted scalings. The errorbars were estimated in the following way : for a given (random) distribution of pinning centres, the measured quantities were averaged over many cycles (forward and backward motion), giving a particular value for the desired parameter. Then we averaged again on different random configurations of pinning centres : this gives a mean value and a typical errorbar for each quantity.

For the pinning force and the Labusch parameter, the errorbars are within the size of the symbols and thus not displayed explicitly in the figure. For both parameters, the numerical results (triangles for κ_L and crosses for F_f^∞) are in good agreement with the predicted scalings of eqs. (33) and (28) : these are illustrated by the dotted lines in the figure, with slope 1 for κ_L and slope 4/3 for F_f^∞ .

Note however the large errorbars involved in the estimation of the memory length, ζ . Indeed, in contrast to the other parameters, the estimated value of ζ fluctuates a lot when different configurations of pinning centres are considered. Good statistics would therefore involve an average over many different (randomly distributed) configurations of defects. Such a procedure requires much more numerical work and goes beyond the purpose of this first stage study, devoted to construct a coherent picture of friction. The numerical results for ζ (circles) are however coherent with the proposed scaling (34), as shown in Fig. 6. The underlying dotted line has a slope $-1/3$, as predicted by eq. (34).

On the other hand, the estimate of the other length δ is obtained to be more precise, since the errorbars involved in the numerical calculations of F_f^∞ and κ_L are both quite small. The numerical results (squares) are seen to follow quite closely the predicted scaling of eq. (36), illustrated in the figure by a line with a slope $-1/3$.

Finally we checked the dependence of the Labusch parameter and of the pinning force as a function of the strength A_p and range R_p of the defects, as given by eqs. (26) and (33). The results are summarized in Figs. 6 and 6. The dependence of κ_L has already been discussed in section 5.1. The results for the friction force F_f^∞ are plotted as triangles in both figures. The predictions of eq. (33) are illustrated as dotted lines of slopes 4/3 for the dependence on A_p and $-5/3$ for the dependence on R_p . Here again, the numerical results are obtained to

be in good agreement with the theoretical predictions. Note that the agreement of the measured F_f^∞ and κ_L with the predicted scalings on A_p and R_p implies necessarily that δ follows the predicted dependence on A_p and R_p as displayed on eq. 36.

Let us emphasize again that we only performed a partial exploration of the parameter space $\{n_p, A_p, R_p\}$, and a more complete numerical work is still to be done. However within the explored window, the trend of *all* the quantities involved in the proposed scenario is obtained to follow the theoretical predictions. Therefore, the numerical results validate with some confidence the proposed picture for the onset of friction.

6 Discussion and conclusions

This work focuses on the onset of friction at the interface between an elastic media and a “rough” surface. In particular, we study how the friction force evolves when the system is pulled *quasistatically* over the surface. Recent experiments have shown that, in this controlled limit, the system exhibits hysteresis and more surprisingly, memory effects [1]. These memory effects are characterized by a *length scale*. While it was known for a while that elasticity is able to produce hysteresis and dissipation, it is shown here that the memory effects originate from the *collective character* of the induced elastic instabilities.

The collective character of quasistatic friction is related to the existence of a correlation length in the system *at equilibrium*, which expresses the balance between the elasticity of the medium and pinning to the defects. When an external constraint is applied to the system, this length is shown to increase linearly with the applied force. This linear increase provides a simple explanation for the *exponential* approach to the stationary static pinning force, as observed numerically and experimentally [1]. Moreover the evaluation of this length allowed us to make some predictions for the dependence of the stationary pinning force and memory length on the microscopic parameters of the system (density, strength and range of the defects of the surface). In particular, it is shown that the value of the *out-of equilibrium* stationary pinning force is fixed by the typical scale for the *fluctuations* of the pinning force *in equilibrium*. The link between the two limits is provided by the analysis of the increase of the correlation length out of equilibrium in the spirit of linear response theory. The predicted scalings are in agreement with the measured numerical results, validating therefore the proposed scenario.

However, our study only focuses on the *quasistatic* motion of the elastic media,

i.e., when any dynamical effect can be neglected. When the system acquires a *finite* velocity, one may expect that the main lines of the proposed scenario will subsist and could provide an interesting alternative route to *dynamical* friction. Such a hope is supported by the analogies between some of our results and the heuristic model of Heslot *et al.* [10]. They show in particular that experimental results for sliding dynamics in the creep controlled regime can be accounted for by a simple model, based mainly on the assumption that the properties of the system depend on its “age”, ϕ , for which a “successful” definition was found to be :

$$\phi \equiv \int_{t_0}^t \exp \left[-\frac{x(t) - x(t')}{D_0} \right] dt' \quad (37)$$

where x is the c.o.m. position of the moving body and D_0 is a “memory length”. In their work, D_0 is assumed to account for plastic flow, but memory effects have been shown in this work to occur even in the elastic limit. This indicates that memory effects do subsist above the depinning threshold. However they have not been discussed on microscopic grounds up to now. The striking analogy between this definition of the age and the exponential decay towards the stationary friction force in our case may be only fortuitous, but this should anyway deserve some more careful investigation. However, such a generalization should be handled with care. In particular, as already emphasized in paragraph 5.2, our understanding of the *creation* of instabilities is only qualitative. In the dynamical case (finite sliding velocity), sound waves induced by instabilities may play a crucial role. These effects are not included in our discussion of the quasistatic limit.

Finally, it is interesting to note that our approach may be generalized to the problem of contact line motion in the quasi-static limit. This problem has been investigated experimentally very recently on nanoscales, using the Surface Force Apparatus (SFA) technique [8]. The experimental results present a striking analogy with those obtained in the experiments on dry friction in the quasistatic limit. In particular, the force acting on the contact line (related to the contact angle of the line) is shown to present hysteresis as a function of the *displacement* of the contact line, defining spatial memory effects characterized by a length scale. Moreover, the approach to the stationary value of the force (*i.e.*, of the advancing or receding contact angle) was obtained experimentally to be *exponential* too. Obviously, all these facts are very similar of those obtained both experimentally and numerically for dry friction. In the case of contact line motion, we expect the motion to be controlled by the competition between the pinning of the contact line to the defects of the surface and on the other hand, the “stiffness” of the line in the direction perpendicular to the motion. If we neglect the energy contribution of the already “wetted” defects, the quasistatic motion of the contact line reduces to a one dimensional problem and it is thus tempting to apply our results. This would lead to a force acting on the contact line, H (denoted

as the “hysteresis of the contact line” in the contact line language) which scales with the number of defects N_p as $H \sim N_p^{2/3}$ (see section 5). This guess is in rather good agreement with both numerical results of ref. [22], where a scaling $H \sim N_p^{0.7}$ was obtained, and even with the experimental results of Di Meglio [19], where a scaling $H \sim N_p^{0.8}$ was measured. Moreover in the latter reference, it was explicitly stressed that avalanches were playing a crucial role in constructing the hysteresis properties. These avalanches are very reminiscent of the instabilities observed in our numerical simulations. Beyond this encouraging agreement, further work is needed to understand more deeply these analogies and characterize more precisely the regime in which these analogies are relevant.

More generally, it is interesting to note that the *nature* of instabilities (here elastic instabilities) does not play a crucial role in the *qualitative* understanding of the onset of friction. The hope would then be to be able to include *plastic effects* into the scenario too, along the same lines as *elastic effects*. Work along these lines is in progress.

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Figure 1 : Graphic resolution of eq. (2) (in the multistable case $2A_p/R_p^2 > \kappa$). The full line is the derivative of the defect gaussian potential. The slope of the straight line is κ , the stiffness of the spring. The intersection gives the solution of eq. (2). Black dots are the stable solutions, the open dot is the unstable solution.

Figure 2 : Hysteresis of the averaged force as a function of the center of mass displacement X_{com} when the elastic chain is pulled back and forth over the defect surface. The parameters are : length of the elastic chain $N = 500$; density of pinning centres $n_p = 0.5$; parameters of the gaussian defect potential $A_p = 0.06$, $R_p = 0.25$.

Figure 3 : Log-linear plot of the averaged force as a function of the center of mass displacement X_{com} for two different densities (top curve $n_p = 0.35$, bottom curve $n_p = 0.2$). The dotted lines are an exponential fit of the averaged force in the large X_{com} limit. The other parameters are $N = 500$, $A_p = 0.06$ and $R_p = 0.25$.

Figure 4 : Plot of the *non-averaged* force as a function of the center of mass displacement, *i.e.*, for one realization of the cycle. The parameters are $N = 500$, $n_p = 0.2$, $A_p = 0.06$ and $R_p = 0.25$.

Figure 5 : Average of the discontinuity in force during an instability, $\langle \Delta F_{inst} \rangle$ as a function of the friction force F_f . The dotted line is linear fit of the numerical points (the few last ones excepted). The slope gives the phenomenological parameter α , while its value at the origin gives δF_0 . For the numerical parameters under consideration ($N = 500$, $n_p = 0.8$, $A_p = 0.06$, $R_p = 0.25$), this gives $\alpha = 0.095$, $\delta F_0 = 1.3$.

Figure 6 : Log-Log plot of the two phenomenological parameters α (squares) and δF_0 (circles), as a function of the density of pinning centres n_p . The straight lines show the predicted scalings : n_p^0 for α and $n_p^{2/3}$ for δF_0 . The length of the chain is $N = 500$ and the parameters for the potential of pinning centres are $A_p = 0.06$, $R_p = 0.25$.

Figure 7 : Log-Log plot of the frequency of instabilities as a function of the density of pinning centres n_p . The straight line has a slope 1/3. The length of the chain is $N = 500$ and the parameters for the potential of pinning centres are $A_p = 0.06$, $R_p = 0.25$.

Figure 8 : Log-Log plot of the different measured quantities as a function of the density of pinning centres n_p . The triangles stand for the Labsuch parameter κ_L ; the crosses for the stationnary static pinning force F_f^∞ ; the circles for the memory length ζ ; and the squares for the length δ . The straight lines indicate

the scalings predicted within the proposed scenario. From bottom to top, the slopes of the lines are $-1/3$, $-1/3$, $2/3$, 1 . The length of the chain is $N = 500$ and the parameters of the potential of pinning centres are $A_p = 0.06$, $R_p = 0.25$.

Figure 9 : Log-Log plot of the stationary pinning force (triangles) and of the Labusch parameter (circles) as a function of the strength of the pinning centres, A_p . The straight lines indicate the predicted scalings : $F_f^\infty \sim A_p^{4/3}$; $\kappa_L \sim A_p^1$. The length of the chain is $N = 500$ and the density of pinning centres $n_p = 0.5$.

Figure 10 : Same as Fig. 6 but for the dependence on the range of the potential, R_p . The straight lines indicate the predicted scalings : $F_f^\infty \sim R_p^{-5/3}$; $\kappa_L \sim R_p^{-1}$. The length of the chain is $N = 500$ and the density of pinning centres $n_p = 0.5$.